Algebraic Ramsey-Theoretic Statements with an Uncountable Flavour

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(various joint works with elements of the set {Ø, Assaf Rinot, 이성협})

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$$6 \to (3)_2^2.$$



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Ramsey theoretic statements are always of the form "however you colour a sufficiently large structure, there will always be monochromatic substructures of some prescribed size".



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Theorem (Schur, 1912)

Whenever we colour the set of natural numbers \mathbb{N} with finitely many colours. there will be two elements x, y such that the set $\{x, y, x + y\}$ is monochromatic.



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Theorem (van der Waerden, 1927)

For every finite colouring of \mathbb{N} and every $k < \omega$ there are two elements a, b such that the set $\{a, a + b, a + 2b, \dots, a + kb\}$ is monochromatic.



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Theorem (Hindman, 1974)

For every finite colouring of \mathbb{N} there exists an infinite set $X \subseteq \mathbb{N}$ such that the set

 $FS(X) = \{x_1 + \dots + x_n | n \in \mathbb{N} \text{ and } x_1, \dots, x_n \in X \text{ are distinct} \}$

(the set of finite sums of elements of X) is monochromatic.

Definition

Let *S* be a commutative semigroup and let θ , λ be two cardinal numbers. The symbol $S \to (\lambda)_{\theta}^{FS}$ will be used to denote the following statement: Whenever we colour the semigroup *S* with θ colours, there will be a set $X \subseteq S$ with $|X| = \lambda$ such that FS(X) is monochromatic.



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Thus Hindman's 1974 theorem from the previous slide simply asserts that $\mathbb{N} \to (\aleph_0)_n^{\mathrm{FS}}$ for every finite *n*. In fact, utilizing the tools from algebra in the Čech–Stone compactification one can prove the following.



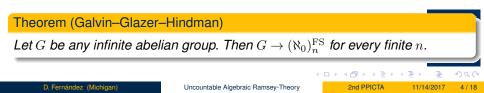
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Theorem (Galvin-Glazer-Hindman)

Let *G* be any infinite abelian group. Then $G \to (\aleph_0)_n^{FS}$ for every finite *n*.

It is natural to ask ourselves whether it is possible to play with the parameters θ, λ in the statement $G \to (\lambda)_{\theta}^{FS}$. In other words, try out an infinite number of colours, or try to increase the size of the monochromatic FS-set.



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Proposition

If *G* is any infinite abelian group, then $G \nleftrightarrow (\aleph_0)_{\aleph_0}^{FS}$.



Let *G* be any uncountable abelian group. Then $G \not\rightarrow (\aleph_1)_2^{FS}$.



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Once again, *S* is a commutative semigroup and θ , λ are cardinals. The symbol $S \to (\lambda)_{\theta}^{FS}$ denotes the statement that whenever we colour *S* with θ colours, there will be a set $X \subseteq S$ with $|X| = \lambda$ such that FS(X) is monochromatic.



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Once again, S is a commutative semigroup and θ , λ are cardinals. The symbol $S \to [\lambda]_{\theta}^{FS}$ denotes the statement that whenever we colour S with θ colours, there will be a set $X \subseteq S$ with $|X| = \lambda$ such that FS(X) avoids at least one colour.



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Once again, *S* is a commutative semigroup and θ , λ are cardinals. The symbol $S \nleftrightarrow [\lambda]_{\theta}^{FS}$ denotes the statement that there exists a colouring of *S* with θ colours such that for every $X \subseteq S$ with $|X| = \lambda$, FS(X) is panchromatic.

Thus,

Theorem (F.B., 2015)

Let *G* be any uncountable abelian group. Then $G \not\rightarrow [\aleph_1]_2^{FS}$.

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Theorem (Milliken, 1978)

Suppose that *G* is a group such that $|G| = \kappa^+ = 2^{\kappa}$ for some cardinal κ . Then $G \nleftrightarrow [\kappa^+]_{\kappa^+}^{FS_2}$

(Where $FS_n(X) = \{x_1 + \dots + x_n | x_1, \dots, x_n \in X \text{ are distinct}\}$, so that $FS(X) = \bigcup_{n \in \mathbb{N}} FS_n(X)$.)



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(However, it is consistent with ZFC that $2^{\kappa} > \kappa^+$ for every infinite cardinal κ .)



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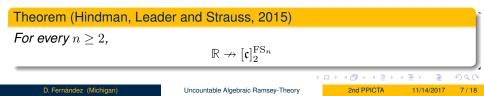
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Theorem (Komjáth and independently D. Soukup and W. Weiss)

For every $n \geq 2$,

$$\mathbb{R} \not\rightarrow [\omega_1]_2^{\mathrm{FS}_n}.$$



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Theorem (Komjáth and independently D. Soukup and W. Weiss)

For every $n \geq 2$,

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Remark (D. Soukup and W. Weiss)

By a theorem of Shelah, it is consistent with ZFC (modulo a large cardinal hypothesis) that $\mathbb{R} \not\rightarrow [\omega_1]_3^{FS_n}$ fails for every $n \ge 2$.



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Let G be any (uncountable) abelian group. Then $G \not\rightarrow [\omega_1]_{\omega}^{FS}$.



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Theorem (F.B. and Rinot, 2016)

It is consistent with ZFC (by assuming $\mathbf{V} = \mathbf{L}$ plus the nonexistence of inaccessible cardinals) that $G \not\rightarrow [\omega_1]_{\omega_1}^{FS}$ holds for every uncountable abelian group G.



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Theorem (F.B. and Rinot, 2016)

Modulo a large cardinal hypothesis (more specifically, the existence of an ω_1 -Erdős cardinal), it is consistent with ZFC that $\mathbb{R} \nrightarrow [\omega_1]_{\omega_1}^{FS}$ fails.

Let *G* be any (uncountable) abelian group. Then $G \nleftrightarrow [\omega_1]^{FS}_{\omega}$.



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Let *G* be any (uncountable) abelian group. Then $G \not\rightarrow [\omega_1]^{FS}_{\omega}$.

Theorem (F.B. and Rinot, 2016)

If *G* is an abelian group of cardinality \beth_{ω} , then $G \to [|G|]^{FS_n}_{\omega}$ (in particular, $G \to [\omega_1]^{FS_n}_{\omega}$) for all $n \in \mathbb{N}$.



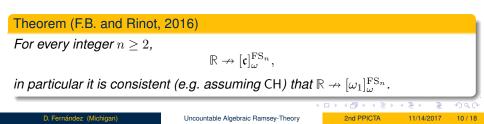
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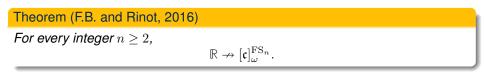
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For every integer $n \geq 2$,

$$\mathbb{R} \nrightarrow [\mathfrak{c}]^{\mathrm{FS}_n}_{\omega}.$$

Theorem (F.B. and Rinot, 2016)

If c is a successor cardinal (e.g., assuming CH), then

$$\mathbb{R} \nrightarrow [\mathfrak{c}]_{\omega_1}^{\mathrm{FS}_n},$$

for every integer $n \geq 2$.



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for every integer $n \geq 2$.

Theorem (F.B. and Rinot, 2016)

Modulo a large cardinal hypothesis (concretely, the existence of a weakly compact cardinal), it is consistent with ZFC that $\mathbb{R} \not\rightarrow [\mathfrak{c}]_{\omega_1}^{\mathrm{FS}_n}$ fails for every integer $n \geq 2$.

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The class of cardinals κ for which every abelian group *G* of cardinality κ satisfies $G \not\rightarrow [\kappa]^{FS_n}_{\kappa}$ for all $n \geq 2$, includes:

- $\kappa = \aleph_1, \aleph_2, \dots, \aleph_n, \dots$; in fact, every successor of a regular cardinal,
- every κ such that $\kappa = \lambda^+ = 2^{\lambda}$,
- every regular uncountable κ admitting a nonreflecting stationary set,
- consistently with ZFC, every regular uncountable cardinal κ .



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Recall that we mentioned that $G \nrightarrow (\omega)^{FS}_{\omega}$ for every infinite abelian group G.



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Recall that we mentioned that $G \not\rightarrow (\omega)^{\text{FS}}_{\omega}$ for every infinite abelian group G.

Theorem (Komjáth 2016)

Given any cardinal κ and any $n \in \mathbb{N}$, there exists a sufficiently large λ (in fact, it suffices to take $\lambda = (\beth_{2^{n-1}-1}(\kappa))^+$) such that $\mathbb{B}(\lambda) \to (n)_{\kappa}^{\mathrm{FS}}$. (Here $\mathbb{B}(\lambda)$ denotes the unique (up to isomorphism) Boolean group of cardinality λ , whose most friendly incarnation is $([\lambda]^{<\omega}, \Delta)$.)



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Theorem (Komjáth 2016)

Given any κ and any $n \in \mathbb{N}$, there exists a sufficiently large cardinal λ (in fact, we can take $\lambda = (\beth_{2^{n-1}}(\beth_{2^{n-1}-1}(\kappa)^+))^+)$ such that, for any colouring of $\mathbb{B}(\lambda)$ with κ colours, we can find elements $x_{\alpha,i}$ ($\alpha < \kappa, i < n$) such that the set

$$\{x_{\alpha_0,i_0} + \dots + x_{\alpha_k,i_k} | \alpha_1, \dots, \alpha_k < \kappa \wedge i_0 < i_1 < \dots < i_k < n\}$$

is monochromatic. We denote this property with the symbol $\mathbb{B}(\lambda) \to (\kappa \times n)_{\kappa}^{\mathrm{FS}_{\mathrm{matrix}}}$.

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Theorem (Carlucci, 2017)

Given an infinite cardinal κ and positive integers c, d, there exists a λ such that, for every abelian group G of cardinality λ , it is the case that for every c-colouring of G there exists $H \subseteq G$ with $|H| = \kappa$ and $a, b \in \mathbb{N}$ such that the set

$$\bigcup_{n \in \{a, a+b, a+2b, \dots, a+db\}} FS_n(H)$$

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$$\bigcup_{n \in \mathrm{FS}(\{a_1, \dots, a_d\})} \mathrm{FS}_n(H)$$

is monochromatic.

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Theorem (F.B. and Lee, 2017)

Given κ , let $\lambda = \beth_1(\kappa)^+ = (2^{\kappa})^+$. Then for every abelian group *G* of cardinality λ , it is the case that

 $G \to (2)^{\text{FS}}_{\kappa}.$



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The upper bound from the previous theorem is optimal. More concretely,

 $\mathbb{B}(2^{\kappa}) \nrightarrow (2)^{\mathrm{FS}}_{\kappa}.$



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Theorem (F.B. and Lee, 2017)

Given κ , let $\lambda = (2^{\kappa})^+$. Then for every abelian group *G* of cardinality λ , it is the case that

$$G \to (\kappa \times 2)^{\mathrm{FS}_{\mathrm{matrix}}}_{\kappa}$$

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An *n*-adequate pattern is a sequence of *n* elements $\langle x_1, \ldots, x_n \rangle \in \bigoplus \mathbb{Z}$ such that for some fixed finite sequence *s* of nonzero integers, it is the case that

$$NZ[FS(\{x_1,\ldots,x_n\})] = \{s\},\$$

where NZ(x) denotes the sequence of non-zero entries of x.



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For example, the sequence $\langle (1, -1, 0), (0, 1, -1) \rangle$ is a 2-adequate pattern.



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For example, the sequence $\langle (1, -1, 0), (0, 1, -1) \rangle$ is a 2-adequate pattern.

Proposition (F.B. and Lee, 2017)

The following are equivalent:

- There exists an *n*-adequate pattern,
- for every κ there exists a λ such that every abelian group G with |G| = λ satisfies G → (n)^{FS}_κ.

We will use the symbol $G \not\rightarrow (\lambda)^+_{\theta}$ to denote the statement that there exists a colouring $c: G \longrightarrow \theta$ such that for every $X \subseteq G$ satisfying $|X| = \lambda$, the set X + X cannot be monochromatic.



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All of the FS_n results of myself and Rinot mentioned previously still hold if we replace FS_2 with + (because $X + X = FS_2(X) \cup 2X$).



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Question (Owings, 1974)

Is it the case that $\mathbb{N} \not\rightarrow (\omega)_2^+$?



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Theorem (Hindman, 1979)

 $\mathbb{N} \nrightarrow (\omega)_3^+.$

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Theorem (Hindman, Leader and Strauss, 2015)

It is consistent with the ZFC axioms (more concretely, it follows from $\mathfrak{c} < \aleph_{\omega}$) that $\mathbb{R} \nrightarrow (\omega)_k^+$ for some finite k.



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Theorem (Komjáth, Leader, Russell, Shelah, D. Soukup and Vidnyánszky, 2017)

Modulo large cardinals (more concretely, assuming the existence of a measurable cardinal), it is consistent that $\mathbb{R} \to (\omega)_r^+$ for all finite r.

